Drift modes in bounded two-ion plasmas with non-Maxwellian electronic distributions

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Abstract

For a two-ion plasma system the electrostatic drift modes have been discussed as it is affected by the presence of non-Maxwellian electronic distributions, namely kappa and Cairns velocity distributions. Analytical solution of the eigenvalue problem is derived for the global drift modes by considering a cylinderical bounded plasma and the corresponding eigen frequency has been obtained. For the D-T plasmas, the eigen frequency has a larger value for the Cairns distribution, intermediate for Maxwellian and smallest for kappa distributed electrons. The influence of magnetic shear on wave propagating has also been examined for a system with absorbing boundaries. It is observed that in the linear limit the perturbed potential distribution is not affected by non-Maxwellian nature of electronic distributions, however the frequency of the drift wave shows a significant deviation from its Maxwellian counterpart.

Introduction

The ordinary plasmas are composed of electrons and single ions, however many of interesting plasmas consist by two-ion species, e.g. the fusion fuel contains D-T (Deuterium and Tritium) ions, the pinch devices constituted by Ne-Ar (Neon and Argon) and the solar corona which is a composition of H-He (Hydrogen and Helium) ions.

Drift modes are one of the most important plasma waves, which are induced by minimal-scale instabilities correlated with drift motions. The drift motions are induced by density and temperature inhomogeneities and the resulting eigenmodes are called drift waves. Usually, the most significant drift waves have real frequencies having a value which is two orders of magnitude less than the ion cyclotron frequency. The study of such low-frequency wave modes, induced by the gradient of plasma density and shear flows, began almost four decades ago. The drift modes can create instability in the system and participate in particle and energy transport. Analysis of low-frequency waves in cylindrical laboratory plasmas with definite scale density gradients, has also found a lot of interest in recent years.

In Ref.[1], the authors considered cylindrical bounded two-ion plasmas and solved the eigenvalue equations analytically for the drift mode. The particle distribution functions (VDFs) as observed in various satellite quests in astrophysical and space plasmas show significance deviation from Maxwellian. It is indicated that VDFs are quasi-Maxwellian at thermal velocities, and exhibit non-Maxwellian at higher speeds. These nominated, that in the magnetospheres of Saturn, Uranus, Mercury, Earth and in the solar wind non-Maxwellian plasmas have been reported in various studies. Such Lorentzian or kappa distributions are specified by spectral index κ such that in the large κ limit Maxwellian distribution is achieved. Almost two decades ago, another non-Maxwellian distribution called the Cairns distribution[2] was successfully used to describe the solitary electrostatic waves, on the ion time scale, detected by different satellite missions such as the Freja and Viking satellites. In Refs.[3, 1] linear electrostatics drift modes are studied in bounded (single and two-ion) plasmas having Maxwllian electron distribution. Here we discuss the linear electrostatics drift modes in bounded two-ion plasma having non-Maxwllian electronic distributions. The corresponding eigen value problems have solved and the effects of non-thermality as well as magnetic shear have been discussed.

Basic equations

The equilibrium qausi-neutrality condition for such plasma consisting of ions a, b and electrons, reads

$$n_{a0}+n_{b0}=n_{e0} \tag{1}$$

The ion equation of motion for j^{th} species can be writen as

$$m_j n_j\left(\partial_t+\mathbf{v}_j\cdot\nabla\right)\mathbf{v}_j=n_j q_j\left(\mathbf{E}+\mathbf{v}_j\times\mathbf{B}_0\hat{z}\right), \tag{2}$$

where j=a,b for two-ions and $\mathbf{E}$ = -$\nabla\phi$ is the electric field and all other terms have their usual meaning. The linearized version of ion continuity equation can be written as

$$\partial_t n_{j1}+n_{j0}\left(\nabla_\perp\cdot\mathbf{v}_{j1\perp}\right)+\left(\nabla_\perp n_{j0}\right)\cdot\mathbf{v}_{j1\perp}=0 \tag{3}$$

By finding $\mathbf{v}_{j1\perp}$ from Eq.(2) and using in Eq.(3), we will get

$$\partial_t\left(n_{a1}+n_{b1}\right)-\left(\frac{n_{a0}}{\Omega_a}+\frac{n_{b0}}{\Omega_b}\right)\frac{1}{B_0}\nabla_\perp^2\partial_t\phi_1+ \nabla_\perp n_{e0}\cdot\frac{1}{B_0}\hat{z}\times\nabla_\perp\phi_1 -\frac{1}{B_0}\left(\frac{\nabla_\perp n_{a0}}{\Omega_a}+\frac{\nabla_\perp n_{b0}}{\Omega_b}\right)\cdot\nabla_\perp\partial_t\phi_1=0 \tag{4}$$

For electrons we have assumed two different non-Maxwellian density profiles, namely the kappa and Cairns distributions. In the former case, the perturbed electron density is given as

$$n_{e1}=n_{e0}\left[1-\frac{e\phi_1}{\left(\left(\kappa-\frac{3}{2}\right)T_e\right)}\right]^{-\kappa+\frac{1}{2}}, \tag{5}$$

where κ ($>$3/2) is the spectral index measuring the deviation from the Maxellian VDF, such that by increasing the value of κ, the kappa distribution approaches to its Maxwellian counterpart. For the case of Cairns VDF,we have [2]

$$n_{e1}=n_{e0}\left[1-\beta\frac{e\phi_1}{T_e}+\beta\left(\frac{e\phi_1}{T_e}\right)^2\right]\exp\left[\frac{e\phi_1}{T_e}\right], \tag{6}$$

where $\beta = 4\Gamma/(1+3\Gamma)$. For weak perturbation the electronic density can be expanded as

$$n_{e1}=n_{e0}\left(\alpha_1\Phi+\alpha_2\frac{\Phi^2}{2}\right) \tag{7}$$

with $\Phi = e\phi_1/T_e$, for linear analysis $\alpha_2\to 0$ and

$$\alpha_1=\begin{cases}1-\beta & \text{(Cairns)}\\ \frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}} & \text{(kappa)}\end{cases}$$

thus, in the linear limit, we have

$$n_{e1}=n_{e0}(\alpha_1\Phi) \tag{8}$$

For two-ions (a and b) system the perturbed form of Poisson's equation reads

$$\nabla^2\phi_1=4\pi e\left(n_{e1}-n_{a1}-n_{b1}\right) \tag{9}$$

$$\lambda_{De}^2\nabla^2\partial_t\Phi-\alpha_1\partial_t\Phi+\left(\frac{n_{a0}}{\Omega_a}+\frac{n_{b0}}{\Omega_b}\right)\frac{T_e}{n_{e0}eB_0}\nabla_\perp^2\partial_t\Phi- \frac{T_e}{eB_0}\frac{\nabla_\perp n_{e0}}{n_{e0}}\cdot\hat{z}\times\nabla_\perp\Phi+ \frac{T_e}{n_{eo}eB_0}\left(\frac{\nabla_\perp n_{a0}}{\Omega_a}-\frac{\nabla_\perp n_{b0}}{\Omega_b}\right)\cdot\nabla_\perp\partial_t\Phi=0, \tag{10}$$

For Golbal drift mode in a cylinder we get the potential as

$$\Phi(r)=cr^p{}_1H_1\left[\frac{a_0^2b_0^2}{4}+\frac{p}{2},1+p,\frac{r^2}{a_0^2}\right] \tag{11}$$

where p is the poloidal mode number.

Effect of magnetic shear on potential distribution

For our analysis we consider a shear magnetic field as follows

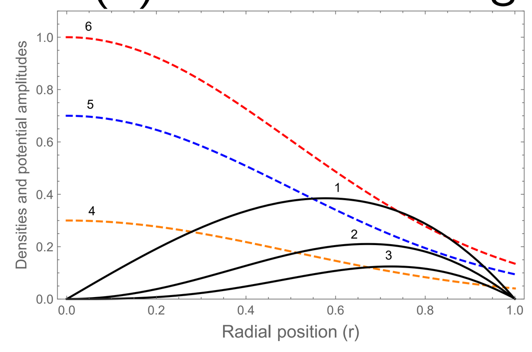
$$\mathbf{B}=B_0\left(\hat{z}+\frac{x}{L_s}\hat{y}\right),$$

where L_s is the shear scale length. We get the potential profile in case of shear magnetic field as

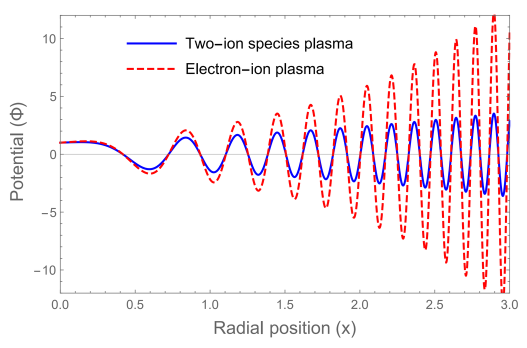
$$\Phi(x)=c_0\exp\left(-\frac{ic_{sa}x^2}{2\rho_{sa}v_* L_s}\sqrt{N_2/N_1}\right) \times\exp\left[\frac{x}{2N_1\rho_{sa}^2}\left(\frac{v_{*a}}{\Omega_a}+\frac{v_{*b}}{\Omega_b}\right)\right] \tag{12}$$

Results and Conclusion

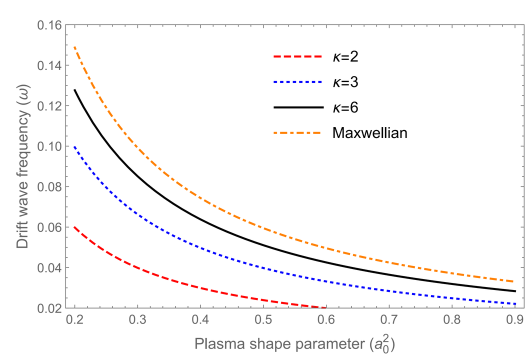
Figures (a) for the case of unsheared magnetic field while (b) for sheared magnetic field



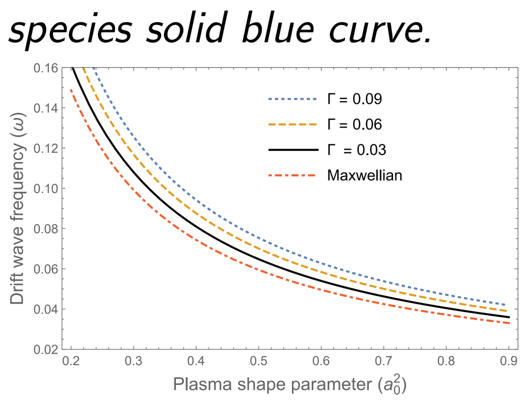
(a) Radial potential profiles presenting the amplitude $\Phi(r)$ in curves 1-3. The curves 4, 5 and 6 show, respectively the equilibrium density profiles of ions a, b and electrons.



(b) *The potential profile for (a) single ion plasma red dashed curve and (b) two-ion species solid blue curve.*



(c) Variation of frequencies of the drift waves in a D-T plasma for different values of κ against plasma shape parameter a_0^2.



(d) *The frequencies of the drift waves for D-T plasma for different values of Γ (for Cairns distributed electron) against plasma shape parameter a_0^2.*

Conclusion

- We have studied drifts modes in two-ion plasmas having non-Maxwellian electronic distributions.
- We have seen that the potential vanishes at center and at the boundaries of the cylinder.
- The Maxwellian distribution achieve by increasing the value of κ, but in case of Cairns distribution it is achieve by when $\Gamma\to 0$.
- Magnetic field geometry plays a significant role.
- The oscillatory behavior in the potential amplitude is due magnetic shear, while the increase of the potential amplitude is due to the density gradient.

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